From Competition to Coordination: What Happens after a Promotion Tournament?*

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Abstract

Firms value individual worker output and the joint output of teams of workers. However, compensation mechanisms which incentivize high effort across employees may not necessarily promote coordination between those employees. In particular, firms often incentivize individual effort through promotion tournaments. Tournaments induce competition and bolster private output but may negatively affect future coordination between competing workers. Previous research shows how pay disparity induces coordination, but the results may change when a tournament determines who receives the higher wage. In this paper, I consider how competition between agents prior to a promotion affects coordination between those agents after a promotion. Essentially, I explore how firms balance the tradeoff between incentivizing individual effort and facilitating coordination.

Keywords: group behavior, contests, experiment, personnel economics

JEL: C72, C91, C92, M54

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1 Introduction

Firms value individual worker output and the joint productivity of teams of workers. They design incentives which maximize each employee's effort and organize such that teams of workers perform effectively together. Firms often facilitate coordination by assigning distinct roles and responsibilities to different workers. This usually involves promoting some to relatively higher levels of authority and corresponding compensation. Previous research (Winter, 2004) shows how firms optimally induce coordination by paying one randomly assigned worker a higher wage. Real-world firms often promote based on particular criteria rather than via random assignment. Many firms promote based on some measure of relative performance in which workers compete for a higher wage. Such promotion tournaments induce competition between coworkers and incentivize high effort from each individual (Lazear & Rosen, 1981). How does competition between workers prior to a promotion affect coordination between those workers following the promotion? In this paper, I unite two influential strands of literature in personnel economics to study how firms balance the tradeoff between incentivizing effort and facilitating coordination.

I study a two-stage game in which two agents compete in a tournament for promotion and subsequently work together on a joint project. In the first stage, a firm designs a schedule of wages and agents expend costly effort to produce output. After observing each agent's output, the firm promotes the highest-producing worker, and the promotee earns the right to a higher wage. In the second stage, agents work on a joint project where project completion depends on the input of both agents. The firm only pays agents if they complete the project. The model captures how firms conducting internal promotions optimally design incentives which induce competition between coworkers without undermining future joint productivity and vice versa.

This paper builds on various strands of literature on personnel economics and organizations. First, I extend the understanding of promotion tournaments. Lazear & Rosen (1981) first models a rank-order tournament in which agents expend effort to produce noisy output, and the agent with the highest output earns the highest wage. The authors frame different wage levels as different tiers in the organizational hierarchy and characterize the tournament as a competition for internal promotion. They discuss how firms utilize promotion and the competition it induces to incentivize effort. Extensions explore promotion tournaments with industrial politics (Lazear, 1989) or external recruitment (Chan, 1996). A related literature models how influence activities affect promotions (P. Milgrom & Roberts, 1988; P. R. Milgrom, 1988). Empirical studies of this topic primarily compare worker output and firm profit under a rank-order tournament versus a piece-rate compensation scheme (Bull et al., 1987; Bognanno, 2001).

Tournament studies primarily focus on worker behavior. By emphasizing the firm's problem, I explore the other side of the tournament and contribute to a relatively sparse literature on optimal contest design (Orrison et al., 2004; Sheremeta, 2011).

The above literature largely focuses on the promotion tournament as a one-shot interaction between competing workers. In this paper, I explore the dynamic incentives for firms and for workers when competing peers continue working together after one earns promotion. In studying the dynamic implications of a rank-order tournament, I contribute to a branch of literature investigating dynamics in contests more generally. Previous work explores theoretically and experimentally how contestants cooperate and communicate in finitely or indefinitely repeated contests (Brookins et al., 2021; Deck et al., 2024). Other research (Rosen, 1986; Altmann et al., 2012) models multi-stage tournaments in which stage winners advance to compete against each other while losers get eliminated. Elimination tournaments model the corporate ladder where

winning each stage represents promotion to a higher rung. While these tournaments focus on competition between stage winners, I explore coordination between the winners and losers of a given stage. I consider cases when promotees must continue working with those they previously bested for promotion.

In particular, I consider a version of the coordination game studied by Winter (2004) and Goerg et al. (2010). In this model, agents work together on a joint project where likelihood of project completion depends on the number of agents who choose to work. A firm designs individual wages for each worker contingent on joint project success. In the two-worker version of the model, a firm cost-minimally facilitates coordination by paying one arbitrarily selected worker a higher wage and offering the remaining worker a lower wage. I study a more realistic environment in which firms intentionally select the higher wage earner via a rank-order tournament.

Some previous research on coordination studies how prior interaction in asset markets affects subsequent coordination (Van Huyck et al., 1993; Kogan et al., 2011). Other research explores the impacts of committed communication in repeated coordination games (Avoyan & Ramos, 2023). I expand this literature by exploring how prior competition in a tournament affects coordination. This also builds on studies which explore behavioral spillovers (Bednar et al., 2012) and strategic linkage (Laferrière et al., 2024) across multiple games more generally.

This work most closely resembles a small vein of literature focusing on post-contest behavior and post-contest interaction between winners and losers. Some of this research explores how contestants' emotions and fairness perceptions affect post-contest decisions (Gill & Stone, 2010; Kräkel, 2008). Two experimental papers explicitly investigate the post-tournament interaction between winners and losers (Johnson & Salmon, 2016; McGee & McGee, 2019). I study an entirely different type of post-tournament interaction and provide a theoretical framework which could offer insight into some of these experimental findings.

The results which follow in section 3 elucidate the dynamics of internal promotions. The model shows how a firm's concern for future coordination affects the optimal design of a promotion tournament. It captures how firms balance the tradeoff between incentivizing individual effort and facilitating joint productivity. While the *relative* wages induce competition between agents, the *absolute* wages affect team coordination. I show generally how the optimal degree of wage inequality depends on the particular work environment and the degree of strategic uncertainty between workers. I then analyze the firm's optimal wage schedule for a particular parametrization.

The model and the results provide a nice starting point for further studying the dynamic implications of internal promotions. I conclude the paper with potential directions for future research in light of the existing personnel economics literature.

2 Model

Consider a two-stage game between two agents ($i = 1, 2$) and one Firm. In Stage 1, agents compete in a tournament for a promotion. Given some wages, agents expend effort to produce some noisy output, and the Firm promotes the highest producer. In Stage 2, agents coordinate to complete a joint project. Project completion depends on both agents completing an individual project task which requires costly effort. The following timeline summarizes the game:

Stage 1 (Promotion): Agents compete for promotion

1. Firm sets wages

2. Agents choose effort
3. Firm promotes one worker

Stage 2 (Project): Agents coordinate on a joint project

1. Each agent assigned individual project task
2. Agents decide to work or shirk
3. Firm pays wages if project is completed

2.1 Stage 1: Promotion

In Stage 1, the Firm designs and conducts a rank-order tournament and promotes one of the two agents. At the end of the tournament, one agent becomes the (L)eader which entitles him to L's wage. The remaining agent becomes the (S)ubordinate and earns the corresponding wage.

First, the Firm posts and commits to a wage schedule $v = (v_L, v_S)$. v_L represents L's wage while v_S represents S's wage. I assume $v_L \geq v_S$ and non-negative wages $v_L \geq 0$ and $v_S \geq 0$. As I describe below, the Firm pays out these wages contingent upon the outcome of Stage 2.

Both agents observe the full wage schedule. Then each agent i chooses effort $a_i > 0$ at cost $c(a_i)$ to produce $y_i = a_i + \varepsilon_i$ where ε_i represents luck randomly drawn and identically distributed for each agent. I assume symmetric cost functions for the agents with $c'(a_i) > 0$ and $c''(a_i) > 0$. The Firm then observes each agent's output y_i and promotes the agent who produces the most. Given the noisy output, each agent possesses some probability P_i of out-producing the other agent and earning promotion. Agent's don't observe each other's effort level or output.

Ex ante, P_i depends on relative output which depends on relative effort and luck. Under the rules of a rank-order tournament, agent i earns promotion with probability

$$P_i = Prob(y_i > y_j) = Prob(a_i + \varepsilon_i > a_j + \varepsilon_j) = Prob(\varepsilon_j - \varepsilon_i < a_i - a_j) = F(a_i - a_j) \qquad (1)$$

where F represents the CDF $(\varepsilon_j - \varepsilon_i)$. I assume a mean zero uniform distribution.

Finally, the Firm assigns roles L and S, and agents progress to the next stage. Agents only receive payment after the second stage. They invest costly effort in the tournament for the right to earn the higher wage in the future. The Firm, however, earns profit from the first-stage tournament. This assumes tournament promotion incentivizes productive output for the Firm. Assuming prices equal 1, the Firm earns expected first-stage profits

$$E[\pi_1] = E[y_1 + y_2] \qquad (2)$$

The Firm maximizes first-stage profit by inducing competition and incentivizing high individual effort and output. However, the firm also values profits in the second stage and designs its wage schedule with this tradeoff in mind.

2.2 Stage 2: Project

In Stage 2, L and S work together on a joint project. The Firm first assigns individual project tasks, and each agent chooses whether to work on his task ($d_i = 1$) at cost $m_L = m_S = m > 0$ or to shirk ($d_i = 0$) at zero cost. If $d_i = 1$, the agent completes his project task with certainty. If $d_i = 0$, he may still complete his project task with probability α. Success in the joint project requires coordination between agents. Joint project completion only occurs if both agents complete their individual project tasks, and payoffs depend only on the outcome of the

joint project. Given a vector of work decisions $d = (d_L, d_S)$, agents complete the joint project with probability

$$q = \begin{cases} 1 & \text{if} \quad d = (1,1) \\ \alpha & \text{if} \quad d = (1,0) \text{ or } d = (0,1) \\ \alpha^2 & \text{if} \quad d = (0,0) \end{cases} \tag{3}$$

Each agent earns the wage corresponding to his role only if the pair jointly completes the project. Otherwise, they both earn zero. The table below summarizes the expected payoffs for L and S given their second-stage work decisions. Let V_L represent L's expected payoffs within the table, while V_S represents S's expected payoffs within the table.

		S	
		$d_S = 1$	$d_S = 0$
L	$d_L = 1$	$v_L - m, v_S - m$	$\alpha v_L - m, \alpha v_S$
	$d_L = 0$	$\alpha v_L, \alpha v_S - m$	$\alpha^2 v_L, \alpha^2 v_S$

The Firm's second-stage profits also depend on project completion. If agents successfully complete the project, the Firm earns D and pays the wages $(v_L + v_S)$ committed at the start of Stage 1. Otherwise, the Firm earns nothing. Therefore, the Firm earns expected Stage 2 profit:

$$E[\pi_2] = q(D - v_L - v_S) \tag{4}$$

The Firm and both agents ultimately optimize respective total payoffs over Stage 1 and Stage 2. Ex ante, each agent i earns the following expected two-stage payoff given some wage schedule:

$$E[u_i] = P_i V_L + (1 - P_i) V_S - c(a_i) \tag{5}$$

While v_L and v_S represent the actual wages, V_L and V_S represent the effective wages. The effective wages capture the expected payoffs in the second stage given anticipated second-stage decisions. I call the difference in actual wages the wage dispersion or spread, and I call the difference in effective wages the effective wage dispersion or spread.

The Firm designs incentives to maximize expected two-stage profit:

$$E[\pi] = E[y_1 + y_2] + q(D - v_L - v_s) \tag{6}$$

In total, the Firm simply earns the sum of profits from the Stage 1 tournament and the Stage 2 project. Consequently, the the Firm's problem involves designing an optimal wage schedule $v^* = (v_L^*, v_S^*)$ which incentivizes high effort and output in the first stage while also encouraging coordination and joint project completion in the second stage.

3 Equilibrium

In this section, I characterize the Subgame Perfect Equilibria of the above game. Given D, b, γ, and m, an SPE specifies the Firm's optimal wage schedule $v^* = (v_L^*, v_S^*)$, each agent's optimal effort a_i^*, and agents' optimal second-stage work decisions d_L^* and d_S^* for every possible wage.

3.1 Stage 2 Nash Equilibria

Solving via backward induction, first consider L and S in Stage 2. L chooses d_L to maximize V_L given v_L. Simultaneously, S chooses d_S to maximize V_S given v_S.

Consider L's problem. Given $d_S = 0$, $d_L = 1$ if $\alpha v_L - m \geq \alpha^2 v_L$. Otherwise $d_L = 0$. Therefore, L chooses $d_L = 1$ if

$$v_L \geq \frac{m}{\alpha(1-\alpha)} \equiv w_L \tag{7}$$

Given $d_S = 1$, $d_L = 1$ if $v_L - m \geq \alpha v_L$. Otherwise $d_L = 0$. Therefore, L chooses $d_L = 1$ if

$$v_L \geq \frac{m}{1-\alpha} \equiv w_S \tag{8}$$

Since $w_L > w_S$, $d_L = 1$ is a dominant strategy if $v_L \geq w_L$. S faces a symmetric problem. The Nash Equilibria of the Stage 2 game and the corresponding payoffs depend on v_L and v_S. If $v_L \geq w_L$ and $v_S \geq w_S$, $d^* = (1,1)$ and agents complete the joint project with certainty. According to Winter (2004), the wage schedule $w = (w_L, w_S)$ represents the cost-minimizing payment scheme that incentivizes full coordination on working and guarantees project completion. The firm guarantees work from L by offering a sufficiently high wage. Knowing that $d_L = 1$, S requires less compensation to work, and the Firm guarantees coordination with $v_S = w_S$. Therefore, unequal wages optimally induce coordination. Winter (2004) assumes a firm must induce project completion and randomly chooses who receives the higher wage w_L.

If $v_L \geq w_L$ and $v_S < w_S$, then $d^* = (1,0)$. In this case, working is a dominant strategy for L, but since $v_S < w_S$, S shirks even conditional on L working. L completes the individual task with certainty, but S completes the individual task with probability α. Therefore, agents complete the joint project with probability α. If $v_L < w_L$ and $v_S < w_S$, then $d^* = (0,0)$ and agents complete the project with probability α^2.

So far, each range of wages induces a unique Stage 2 NE. If $w_s \leq v_L < w_L$ and $v_S \geq w_S$, then three NE exist. There exist two equilibria in pure strategies: 1) $d^* = (1,1)$ and 2) $d^* = (0,0)$. There also exists one equilibrium in mixed strategies in which $d_L = 1$ with probability $\delta_L = \frac{v_S(\alpha^2-\alpha)+m}{v_s(1-2\alpha+\alpha^2)}$, and $d_S = 1$ with probability $\delta_S = \frac{v_L(\alpha^2-\alpha)+m}{v_L(1-2\alpha+\alpha^2)}$. At this mixed-strategy equilibrium, expected L payoff $V_L = \alpha^2 v_L + (\alpha - \alpha^2)\frac{v_L(\alpha^2-\alpha)+m}{1-2\alpha+\alpha^2}$ and expected S payoff $V_S = \alpha^2 v_S + (\alpha - \alpha^2)\frac{v_S(\alpha^2-\alpha)+m}{1-2\alpha+\alpha^2}$.

3.2 Stage 1 Agent's Problem

Moving back to Stage 1, agents observe wages v_L and v_S and anticipate the Stage 2 outcome and corresponding expected payoffs V_L and V_S. Given these effective wages, agents compete in the Stage 1 tournament for the right to earn V_L in the next stage. V_L and V_S imply an effective wage spread $V_L - V_S$ which represents the expected benefit of promotion. Since $V_L \geq V_S$ for any $v_L \geq v_S$, agents always prefer the role of L in Stage 2. Therefore, assuming wages offer a non-negative expected utility, agents expend costly effort in the tournament to increase their chances of becoming L. Each agent chooses a_i given V_L and V_S to maximize expected two-stage payoff:

$$\max_{a_i} P_i V_L + (1 - P_i)V_S - c(a_i) \tag{9}$$

The optimal a_i equalizes the marginal benefit and marginal cost of an increase in effort:

$$f(a_i - a_j)(V_L - V_S) = \frac{\partial c(a_i)}{\partial a_i} \tag{10}$$

where $f(a_i - a_j) = \frac{\partial P_i}{\partial a_i}$ and f represents the PDF of $(\varepsilon_j - \varepsilon_i)$.

Assuming symmetric agents with symmetric increasing and convex $c(a_i)$, equilibria only include symmetric effort choices (Proof in Appendix A.1.) such that $a_i = a_j = a$. This implies $a_j - a_i = 0$ and the following optimality condition for each agent:

$$f(0)(V_L - V_S) = \frac{\partial c(a)}{\partial a} \tag{11}$$

In equilibrium, since $a_i = a_j = a$, each agent wins the tournament with probability $\frac{1}{2}$. Agents choose optimal effort $a^* \geq 0$ satisfying the above condition as long as $E[u] = \frac{1}{2}(V_L + V_S) - c(a^*) \geq 0$. Otherwise, agents choose $a^* = 0$.

Each agent's utility-maximizing effort decision depends on the effective wage spread $V_L - V_S$ and the degree of noise in output. An increase in the effective wage spread makes the L position more attractive and increases effort. On the contrary, noisier output diminishes agents' influence over the promotion decision and discourages effort.

3.3 Stage 1 Firm's Problem

Finally, the Firm anticipates the future behavior of agents and chooses a wage schedule $v^* = (v_L^*, v_S^*)$ which maximizes profits under certain constraints from the agent's problem. First, $\frac{1}{2}(v_L + v_S) \geq c(a)$ represents an Individual Rationality constraint ensuring that agents are willing to enter the promotion tournament with $a^* \geq 0$. Second, $f(0)(V_L - V_S) = \frac{\partial c(a)}{\partial a}$ represents an Incentive Compatibility constraint which guarantees that agents who expend positive effort in the tournament always choose the optimal utility-maximizing effort. Finally, I restrict the Firm to non-negative wages such that $v_L \geq v_S \geq 0$. Under these constraints, the Firm chooses v_L and v_S to maximize expected two-stage profits:

$$\max_{v_L, v_S} E[y_1 + y_2] + q(D - v_L - v_S) \quad \text{s.t.} \tag{12}$$

$$\frac{1}{2}(V_L + V_S) \geq c(a) \quad (IR) \tag{13}$$

$$f(0)(V_L - V_S) = \frac{\partial c(a)}{\partial a} \quad (IC) \tag{14}$$

$$v_L \geq v_S \geq 0 \tag{15}$$

As I describe in the previous two subsections, wages affect agents' decisions in the first-stage tournament and the second-stage project. In particular, the *relative* wages determine how agents compete for promotion while the *absolute* wages influence joint project work decisions. Since both first-stage output and second-stage project completion increase the Firm's profits, the Firm carefully designs an incentive scheme which balances incentivizing individual effort and facilitating coordination. For example, a wage schedule with a significant wage spread may induce heavy competition for promotion and bolster first-stage output but reduce the probability of joint project completion. The Firm optimizes individual effort and output but sacrifices team

productivity. On the other hand, a wage schedule which offers sufficiently high wages to both agents to guarantee joint project success may undermine the promotion incentives and limit competition. In this case, the Firm optimizes team productivity but sacrifices individual output. The Firm faces a complex problem when designing an incentive mechanism which balances these two elements of profit.

3.3.1 Solving the Firm's Problem

In order to gain further insight into the Firm's problem, I impose additional parametric assumptions. Assume a particular cost of effort $c(a) = \frac{1}{2\gamma}a^2$ where $\gamma > 0$ is a constant. Also assume a distribution of the difference in luck $(\varepsilon_j - \varepsilon_i) \sim U[-b, b]$ with $b > 0$. This distribution implies $f(0) = \frac{1}{2b}$. These assumptions imply closed-form expressions for a^* in terms of V_L and V_S and the corresponding $c(a^*)$. From the agent's problem,

$$a^* = \frac{\gamma(V_L - V_S)}{2b} \tag{16}$$

$$c(a^*) = \frac{1}{2\gamma}\frac{\gamma^2(V_L - V_S)^2}{4b^2} = \frac{\gamma(V_L - V_S)^2}{8b^2} \tag{17}$$

In solving this complex problem, the Firm determines the local solutions to a series of artificially constrained optimization problems and then compares these solutions to find the global optimum. For example, the Firm self-imposes the constraints $v_L \geq w_L$ and $v_S \geq w_S$ and finds the optimal wage schedule. This induces the unique second-stage NE $d^* = (1, 1)$ with $q = 1$. $V_L = v_L - m$ and $V_S = v_S - m$ which permits expressing the Firm's problem entirely in terms of actual wages v_L and v_S. Within this set of wages, changes in v_L and v_S don't effect q. However, manipulating wages in this range affects first-stage incentives. The Firm solves the constrained optimization problem

$$\max_{v_L, v_S} \quad 2\left(\frac{\gamma(v_L - v_S)}{2b}\right) + D - v_L - v_S \quad \text{s.t.} \tag{18}$$

$$\frac{1}{2}(v_L + v_S - 2m) = \frac{\gamma(v_L - v_S)^2}{8b^2} \quad (IR) \tag{19}$$

$$v_L \geq w_L \quad v_S \geq w_S \tag{20}$$

I substitute the IC constraint directly into the Firm's problem, and the IR constraint binds with equality such that the Firm extracts all surplus from the agents. Solving this problem involves first assuming non-binding constraints on v_L and v_S and then verifying. The Firm chooses optimal wage schedule $v^*_{1,1} = (\frac{\gamma}{2} + m + b, \frac{\gamma}{2} + m - b)$ which implies a wage spread of $2b$. This wage schedule satisfies the self-imposed constraints on v_L and v_S if $\frac{m(1-\alpha+\alpha^2)}{\alpha(1-\alpha)} - \frac{\gamma}{2} \leq b \leq \frac{\gamma}{2} - \frac{m\alpha}{1-\alpha}$.

The Firm repeats this procedure for each possible Stage 2 outcome. This provides a menu of options that maximize first-stage profit given some second-stage expected profit. Finally, the Firm selects from this menu whether it prefers inducing second-stage work from both agents, one agent, or neither agent. In doing so, the Firm finds the two-stage profit-maximizing wage schedule.

The optimal second-stage outcome depends on the Firm's environment. Consider when the Firm might select a wage schedule which guarantees full second-stage coordination. This implies a wage spread of at least $w_L - w_S$ and substantial wages for both L and S. The wage spread

incentivizes individual effort from both agents, and the sufficiently high wage level for each worker facilitates coordination. However, high wage levels also increases the Firm's costs. This should be the optimal wage schedule when the Firm values both incentivizing high individual effort and maintaining a productive working relationship between agents in the future.

Consider when the Firm instead prefers only inducing second-stage work from L. This represents a sort of winner-take-all incentive scheme through which the Firm induces competition between both agents but only prioritizes the future productivity of the promotee. The Firm again institutes a wage spread of at least $w_L - w_S$ but perhaps with a low absolute wage for S. The low relative wage and low absolute wage for S cultivates dissension and damages the future working relationship between agents. S shirks on the subsequent joint project while high-earning L works alone. The winner-take-all wage schedule predominantly secures future productivity from only L while still encouraging high effort from both agents during promotion.

Finally, consider when the Firm selects an optimal wage schedule which induces second-stage work from neither agent. In this case, joint team coordination offers little return and the Firm focuses on incentivizing individual effort and keeping wages low. Given this discussion, I now offer three general insights regarding the Firm's optimal wages. The following results assume $\alpha = \frac{1}{2}$. Proposition 1 considers the optimal degree of wage inequality that induces full coordination.

Proposition 1. *(Inequality) If $m < b < \gamma$ and $\gamma < 4m$, then there exists a $\bar{D}$ such that for any $D \geq \bar{D}$, the optimal wage schedule induces project completion with certainty ($d^* = (1,1)$) with wage dispersion $v_L^* - v_S^* \geq w_L - w_S$.*

Proof. First, consider the effect on profit of a small increase in v_S. Given v_L,

$$\frac{\partial \pi}{\partial v_S} = -2(\frac{\gamma}{2b}q) - q \tag{21}$$

Notice $\frac{\partial \pi}{\partial v_S} < 0$ always. Therefore, the Firm always prefers to reduce v_S as low as possible for a given v_L. Consider the effect on profit of a small increase in v_L. Given v_S,

$$\frac{\partial \pi}{\partial v_L} = 2(\frac{\gamma}{2b}q) - q \tag{22}$$

Notice $\frac{\partial \pi}{\partial v_L} > 0$ if

$$\frac{\gamma}{b} > 1 \tag{23}$$

Therefore, if $b < \gamma$, the Firm prefers the highest feasible v_L. From solving the Firm's problem, the optimal wage spread which guarantees $d^* = (1,1)$ is $v_L - v_S = 2b$. If this optimal wage schedule is feasible, then the corresponding wage spread exceeds $w_L - w_S = 2m$ if $b > m$.

If the optimal wage schedule is not feasible, since $b < \gamma$, the Firm finds the optimal wage schedule by setting $v_S = w_S$ and increasing $v_L \geq w_L$ until agent's $E[u] = 0$. This results in a wage spread no smaller than $w_L - w_S = 2m$ as long as the wage schedule $v = (w_L, w_S)$ satisfies $E[u] \geq 0$. Otherwise, the Firm must increase v_S in this range which could result in a wage spread less than $w_L - w_S = 2m$. For $v = (w_L, w_S)$, $E[u] \geq 0$ if $\frac{1}{2}(4m) - \frac{\gamma(2m)^2}{8b^2} \geq 0$. This occurs if $m \leq 4\frac{b^2}{\gamma}$. Since $m < b$, this condition holds for certain when $\gamma < 4m$.

Therefore, the wage schedule which most profitably induces full coordination $d^* = (1,1)$ involves a wage schedule such that $v_L^* - v_S^* \geq w_L - w_S$.

The firm may also consider alternative wage schedules which induce $d^* = (1, 0)$ or $d^* = (0, 0)$. These wage schedules offer expected value of the second-stage project $\frac{1}{2}D$ and $\frac{1}{4}D$ respectively. Therefore, if D is sufficiently high, the firm prefers the wage wage schedule which induces $d^* = (1, 1)$ and guarantees project value of D with certainty in the second stage.

□

Winter (2004) assumes the Firm must incentivize full coordination between agents and finds the optimal incentive-inducing mechanism with a wage spread of $w_L - w_S$. The Firm randomly chooses who receives w_L and w_S, and this unequal wage schedule guarantees project completion at a lower cost than one which pays equal wages. According to Proposition 1, when the Firm selects who receives the higher wage via a promotion tournament, it may prefer a coordination-inducing incentive mechanism with even greater inequality. Expanding the wage gap induces more competition between agents competing for promotion. Therefore, for sufficiently low noise in output such that the Firm values individual effort prior to a promotion, the optimal wage gap which induces coordination may involve greater inequality than Winter (2004) proposes.

Proposition 2 describes the opposite result. In certain cases, the Firm induces full coordination and guarantees project completion with equal wages.

Proposition 2. *(Equality) If $b > \gamma$ and $d^* = (1, 1)$ when $w_s \leq v_L < w_L$ and $v_S \geq w_S$, then there exist values of D for which the Firm induces full coordination and guarantees project completion with equal wages $v_L^* = v_S^* = w_S$.*

Proof. Since $b > \gamma$, we know from Proposition 1 that increasing the wage spread decreases profits. Therefore, the Firm prefers setting v_L and v_S as low as possible.

First, I show that for all D, the Firm prefers equal wages over an alternative wage spread which guarantees full coordination. Since agents coordinate on $d^* = (1, 1)$ when $w_s \leq v_L < w_L$ and $v_S \geq w_S$, profits from the wage schedule $v_M^* = (w_S, w_S) = (2m, 2m)$ are $\pi_M^* = D - 4m$.

The Firm could also induce full coordination $d^* = (1, 1)$ with the wage schedule $v_{1,1}^* = (w_L, w_S) = (4m, 2m)$. This yields profit $\pi_{1,1}^* = 2m(\frac{\gamma}{b}) + D - 6m$. $\pi_{1,1}^* < \pi_M^*$ if $b > \gamma$.

Other alternatives which don't achieve full coordination include $v_{1,0}^* = (4m, 0)$ which yields profit $\pi_{1,0}^* = m(\frac{\gamma}{b}) + \frac{1}{2}D - 2m$ and $v_{0,0}^* = (0, 0)$ which yields profit $\pi_{0,0}^* = \frac{1}{4}D$. The Firm prefers to induce full coordination and guarantee project completion if $D \geq 4m + \frac{m}{2}\frac{\gamma}{b}$.

Therefore, if $b > \gamma$, agents coordinate on $d^* = (1, 1)$ when multiple equilibria exist and $D \geq 4m + \frac{m}{2}\frac{\gamma}{b}$, then the Firm guarantees full coordination in Stage 2 with an optimal wage schedule consisting of equal wages $v_L^* = v_S^* = w_S$.

□

Multiple equilibria exist for the intermediate range of wages when both $w_s \leq v_L < w_L$ and $v_S \geq w_S$. If competing agents both agree to work together following promotion, then the Firm can optimally guarantee joint project completion with equal wages. This describes work environments which reduce the strategic uncertainty that agents will shirk, mitigate the risk of miscoordination and drive workers to coordinate on more efficient equilibria. For example, previous research demonstrates how competition between groups improves coordination and cooperation on more efficient outcomes within groups (Bornstein et al., 2002; Tan & Bolle, 2007; Gunnthorsdottir & Rapoport, 2006). Competition with other groups provides a focal point which reduces strategic uncertainty within a group. Settings with across-group competition may cultivate more efficient coordination between in-group workers and require less dispersion

in wages. This result extends Winter (2004) which suggests unequal wages as the optimal coordination-inducing incentive mechanism.

Finally, Proposition 3 explores S's wage. While relative wages drive competition in the promotion tournament, absolute wages determine second-stage work. Proposition 3 describes particular cases when S earns the lowest possible wage.

Proposition 3. *(Subordinate Wage) If $b > \frac{1}{2}\gamma$ and $m \leq \gamma$, then $v_S^* \leq w_s$. In particular, depending on D, $v_S^* = w_S$ or $v_S^* = 0$.*

Proof. We know from Proposition 1 that if $b > \gamma$, increasing the wage spread decreases profits. Consequently, when $b > \gamma$, the Firm sets v_L and v_S as low as possible. If $b \leq \gamma$ and the optimal wage schedule which induces a particular Stage 2 outcome is unattainable because it doesn't satisfy the artificial constraints on v_L and v_S, the Firm forces the artificial constraint to bind on v_S and sets v_L such that $E[u] \geq 0$. Recall $v_L \geq v_S \geq 0$.

The optimal wage schedule which induces $d^* = (1,1)$ is $v_{1,1}^* = (\frac{\gamma}{2} + m + b, \frac{\gamma}{2} + m - b)$. Attainability of this wage schedule requires $b \leq \frac{\gamma}{2} - 3m$. The optimal wage schedule which induces $d^* = (1,0)$ is $v_{1,0}^* = (\gamma + 2b, \gamma - 2m - 2b)$ which requires $b \leq \frac{\gamma}{2} - 2m$. Finally, the optimal wage schedule which induces $d^* = (0,0)$ is $v_{0,0}^* = (2\gamma + 4b, 2\gamma - 4b)$ which requires $b \leq \frac{\gamma}{2}$.

Therefore, $b > \frac{1}{2}\gamma$ implies unattainability of the locally optimal wage schedule for each Stage 2 outcome. The Firm achieves each local optimum by setting v_S as low as possible within the relevant range. Inducing $d^* = (1,1)$ involves setting $v_S^* = w_S$ while inducing $d^* = (1,0)$ or $d^* = (0,0)$ involves setting $v_S^* = 0$. Which option the firm chooses depends on D.

I must also verify that setting v_S at the minimum within a relevant range is feasible. As in the proof of Proposition 1, this holds if $E[u] \geq 0$. Otherwise, the Firm must increase v_S. $E[u] \geq 0$ in all cases in which v_S is set at the minimum if $m \leq 4\frac{b^2}{\gamma}$. Since $b > \frac{1}{2}\gamma$, this condition holds for certain when $\gamma \geq m$.

□

Proposition 3 describes situations in which the agent who misses out on promotion earns a minimal wage. While the promotee advances upward in the Firm and enjoys high relative and absolute compensation, the subordinate earns nothing or just enough to incentivize future work.

3.4 Comparative Statics

Before analyzing a particular example, I first describe in general some effects of changes in certain parameters. First, consider the degree of noise in output b. As discussed above, greater noise unequivocally decreases equilibrium first-stage effort which reduces Firm profits for a given wage spread. The Firm often compensates agents for the increased noise by increasing the wage spread and restoring some effort. However, since noise diminishes agents' influence over who wins promotion, it also weakens the incentives of the tournament and reduces the Firm's marginal benefit of increasing the wage spread. Conversely, an increase in γ reduces the marginal cost of effort and increases output. This increases profits for the Firm and puts upward pressure on v_L^* and v_S^* without necessarily changing optimal wage dispersion.

The effects of changes in m and α are more complicated. To illustrate these effects, assume the Firm adopts the optimal wage schedule proposed by Winter (2004): $v = (w_L, w_S) = (\frac{m}{\alpha(1-\alpha)}, \frac{m}{1-\alpha})$. This implies a wage spread of $\frac{m}{\alpha}$, first-stage effort $a = \frac{\gamma}{b}\frac{m}{2\alpha}$, second-stage work $d = (1,1)$ and two-stage profits $\pi = \frac{\gamma}{b}\frac{m}{\alpha} + D - \frac{m(1+\alpha)}{\alpha(1-\alpha)}$. Consider an increase in m. Clearly this

increases the wage spread $w_L - w_S$ compensating the agent for a higher cost of second-stage work. Agents' effort increases, but the Firm's costs $w_L + w_S$ also rise. Profits increase if $\frac{\gamma}{b} \geq \frac{1+\alpha}{1-\alpha}$. If $\alpha = \frac{1}{2}$, this requires $\gamma \geq 3b$. Therefore, an increase in m only increases the profitability of a coordination-inducing wage schedule for a sufficiently high ratio of $\frac{\gamma}{b}$.

Now consider an increase in the probability of task completion from shirking α. This makes shirking even more attractive for agents. The wage spread $w_L - w_S$ and the corresponding agent effort decrease. Profits for the firm increase if $\frac{\gamma}{b} \leq \frac{\alpha^2(1+\alpha)(1-2\alpha)}{(\alpha-\alpha^2)^2} - \frac{\alpha^2}{\alpha-\alpha^2}$. Therefore, an increase in α only increases the profitability of a coordination-inducing wage schedule for a sufficiently low ratio of $\frac{\gamma}{b}$. Notice how the ratio $\frac{\gamma}{b}$ influences the Firm's optimal wage schedule for different values of m and α. I explore particular values of $\frac{\gamma}{b}$ in the example below.

An increase in the value of the second-stage project D makes wage schedules which induce a higher probability of project completion more attractive for the Firm. Since b affects first-stage output and D affects second-stage profits, $\frac{D}{b}$ effectively measures the key tradeoff the Firm faces between incentivizing high effort and facilitating coordination. In the next section, I analyze the effects of changes in this ratio more closely.

3.5 Solution and Comparative Statics in $\frac{D}{b}$

In this section, I fully characterize the equilibria of the above game for a particular set of parameters (see Appendix A.2 for more detail on solving the Firm's problem for this example). I assume $\gamma = 4$, $m = 2$ and $\alpha = \frac{1}{2}$ which imply $w_L = 8$ and $w_S = 4$. In this section, I emphasize the effects of changes in $\frac{D}{b}$ on the Firm's optimal incentive design.

I also study the important relationship between γ and b. As discussed above, this relationship between the marginal cost of effort and the degree of noise in output informs how agents respond to wages and how the Firm values incentivizing first-stage effort. From Proposition 1, we know that increasing v_L only increases profit if $2 < b < 4$. Furthermore, from Proposition 3, we know that if $b \geq 2$, either $v_s = 4$ or $v_S = 0$.

Given some wages, agents follow the Stage 2 strategy described above which implies effective wages V_L and V_S (more detail in Appendix A.2). Given effective wages, if $E[u] = \frac{1}{2}(V_L + V_S) - \frac{(V_L - V_S)^2}{2b^2} \geq 0$, agents choose effort $a^* = \frac{2(V_L - V_S)}{b}$. Otherwise, $a^* = 0$. The Firm anticipates agents' strategy and chooses the profit-maximizing wage schedule following the procedure outlined above.

First, consider when $2 < b < 4$. Figure 1 illustrates the profit-maximizing wages for different values of $\frac{D}{b}$. Each colored region specifies an optimal wage schedule which induces a particular effort level and second-stage project outcome. For these values of b, any wage schedule in the range $4 \leq v_L < 8$ and $v_s \geq 4$ is always dominated. In the red region, the Firm prefers the wage schedule $v^* = (4 + \frac{b^2 + b\sqrt{b^2+16}}{2}, 4)$ which induces equilibrium effort $a^* = b + \sqrt{b^2 + 16}$ and second-stage work decisions $d^* = (1, 1)$. In the green region, the Firm prefers the wage schedule $v^* = (4 + 2b^2, 0)$ which induces equilibrium effort $a^* = 2b$ and second-stage work decisions $d^* = (1, 0)$. Finally, in the white region, the Firm prefers the wage schedule $v^* = (8, 0)$ which induces equilibrium effort $a^* = \frac{4}{b}$ and second-stage work decisions $d^* = (0, 0)$.

In the red region, the high absolute wages guarantee coordination and the high wage spread maximizes first-stage effort. $v^*_{1,1}$ offers the highest effective wage spread and effort for all values of $\frac{D}{b}$. However, this wage schedule also requires the highest cost. The benefits outweigh the costs for sufficiently high values of D. As b increases, the Firm raises v^*_L, but noise weakens the

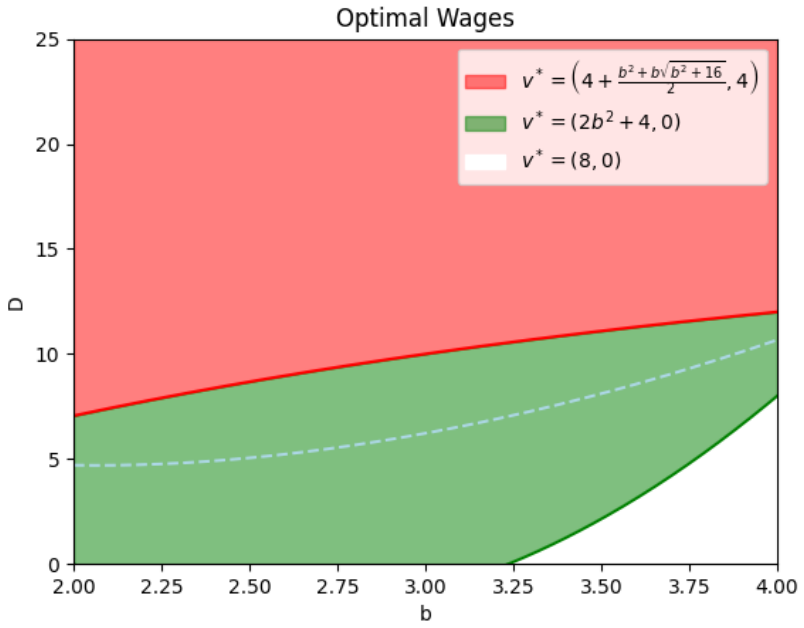


Figure 1: $2 < b < 4$

tournament incentives and diminishes the benefits of increasing the wage spread. Second-stage benefits must increase to outweigh the costs, so the critical value of D increases.

The green region represents the winner-take-all incentive scheme through which the Firm induces competition between both agents but only prioritizes the future productivity of the promotee. The Firm cares so little about S's work in this case that it reduces v_S to zero. $v^*_{1,0}$ induces the highest actual wage spread but only the second-highest effective wage spread and effort level. Again, more noise diminishes the benefit of a higher wage spread, increases the relative cost of raising wages and requires higher D to validate paying absolute wages which induce second-stage work.

Finally, the white region represents cases in which joint team coordination offers little return and the Firm only focuses on incentivizing individual effort and minimizing costs. Agents only earn positive expected utility in this region. For sufficiently low values of the second-stage project and sufficiently high values of noise, the Firm prefers this wage schedule even though it leaves some surplus on the table for the agents.

Under noisier output, incentivizing effort while also facilitating coordination in the red and green regions becomes increasingly costly and less beneficial. However, costs remain fixed in the white region. This explains why the green region gets squeezed for low values of $\frac{D}{b}$. Eventually the winner-take-all incentive scheme offers little savings in cost relative to the full-coordination-inducing mechanism and little added benefit in expected second-stage profit relative to the cost-effective mechanism which guarantees shirking.

Next, consider when $b \geq 4$. Based on Proposition 1, the Firm has no incentive to increase v_L. Due to the high degree of noise, the increase in cost outweighs the gain in profit from the corresponding increase in effort. The Firm prefers to set v_L and v_S as low as possible.

Optimal wages in this case depend on how agents coordinate when multiple equilibria exist.

First, consider the Firm's optimal wages when agents coordinate on $d^* = (1,1)$ or the mixed-strategy equilibrium $d^* = (\delta_L = \frac{8-v_S}{v_S}, \delta_S = \frac{8-v_L}{v_L})$. Consider Figure 2. In the red region, the Firm prefers the wage schedule $v^* = (4,4)$ which induces equilibrium effort $a^* = 0$ and second-stage work decisions $d^* = (1,1)$ due to the coordination of the agents. In the green region, the Firm prefers the wage schedule $v^* = (8,0)$ which induces equilibrium effort $a^* = \frac{4}{b}$ and second-stage work decisions $d^* = (1,0)$. Finally, in the white region, the Firm prefers the wage schedule $v^* = (0,0)$ which induces equilibrium effort $a^* = 0$ and second-stage work decisions $d^* = (0,0)$.

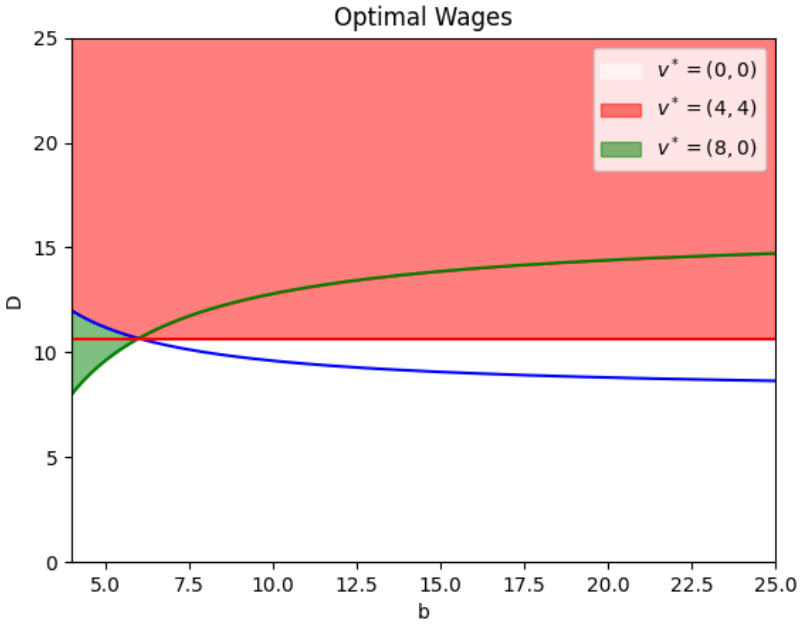


Figure 2: $b \geq 4$ and $d^* = (1,1)$ or $d^* = (\delta_L, \delta_S)$

In this scenario, the Firm only offers a positive wage spread and induces positive effort in the green region for relatively low values of b. Noisy output limits agents' influence over the promotion decision and greatly weakens the incentives of a winner-take-all incentive mechanism. Basically removing the winner-take-all schedule from consideration, the decision between inducing full second-stage working or shirking depends purely on the value of the second-stage project D.

Since agents agree to coordinate on second-stage work for lower wages and the Firm can guarantee project completion at a lower cost, the Firm never prefers a wage schedule in which $v_L \geq w_L$ and $v_S \geq w_S$. Furthermore, agents earn the highest expected utility when they coordinate on working. Proposition 2 describes how coordination on efficient equilibria may result in lesser wage dispersion. It may also increase the expected utility of agents competing for promotion.

Finally, consider the case in which $b \geq 4$ but agents coordinate on $d^* = (0,0)$ if $4 \leq v_L < 8$ and $v_s \geq 4$. Figure 3 depicts optimal wage schedules in this case. In the red region, the Firm prefers the wage schedule $v^* = (8,4)$ which induces equilibrium effort $a^* = \frac{8}{b}$ and second-stage work decisions $d^* = (1,1)$. In the green region, the Firm prefers the wage schedule $v^* = (8,0)$

which induces equilibrium effort $a^* = \frac{4}{b}$ and second-stage work decisions $d^* = (1, 0)$. Finally, in the white region, the Firm prefers the wage schedule $v^* = (0, 0)$ which induces equilibrium effort $a^* = 0$ and second-stage work decisions $d^* = (0, 0)$.

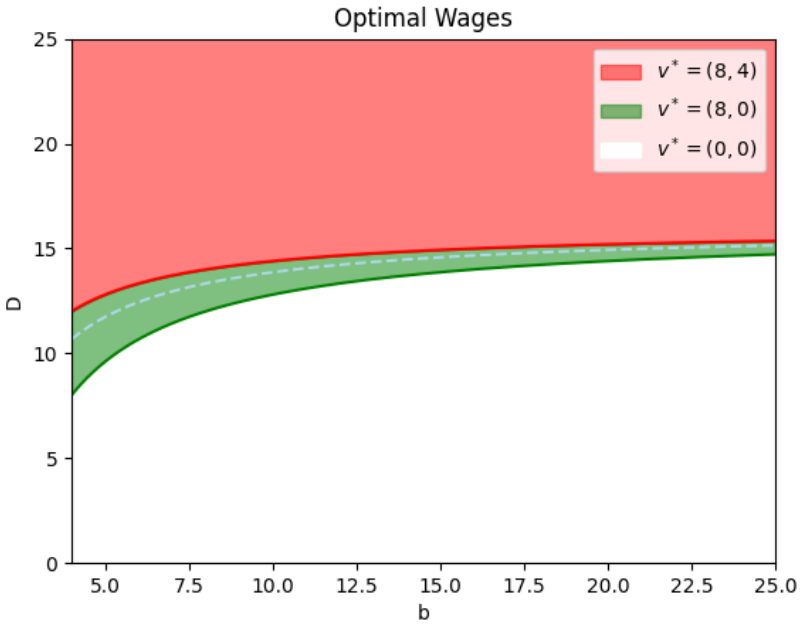


Figure 3: $b \geq 4$ and $d^* = (0, 0)$

This situation captures much of the same intuition as that of Figure 1. However, due to the higher degree of noise, offering positive absolute wages and inducing second-stage work requires even higher Stage 2 project value D. Agents in this case only earn positive utility under $v^*_{1,1}$ and $v^*_{1,0}$ with $v^*_{1,1}$ yielding the highest expected utility.

This specific example illustrates how changes in $\frac{D}{b}$ affect equilibrium wages, effort, and coordination for different values of $\frac{\gamma}{b}$ and provides insight into how firms balance the tradeoff between incentivizing individual effort and facilitating joint productivity in different environments.

4 Tullock Contest

In personnel economics, we generally model competitive internal promotion as a rank-order tournament. However, the rank-order tournament represents only one case of a broader class of contests. In general, a contest describes any situation in which participants irrevocably expend costly resources to affect their chances of winning a prize. Each different contest type specifies particular assumptions regarding competition and winner selection rules. So far I've focused only on the rank-order tournament. In this section, I explore whether results change qualitatively if firms instead promote via a Tullock lottery. I analyze how a Tullock lottery promotion affects competition, post-contest coordination, and optimal wage design.

Consider a version of the two-stage game described above in which the Firm promotes the winner of a Tullock lottery in Stage 1. In this game, the post-contest environment and equilibria

remain unchanged, but the rules and strategies of the first stage differ. Following the traditional setup of the Tullock lottery, I assume non-noisy output and linear costs such that $y_i = a_i$ and $c(a_i) = a_i$ for each agent i. The Firm promotes the agent who wins the Tullock lottery which occurs for agent i with probability

$$P_i = \frac{a_i}{a_i + a_j} \tag{24}$$

As before, each agent i maximizes expected utility:

$$\max_{a_i} P_i V_L + (1 - P_i) V_S - c(a_i) \tag{25}$$

where now $P_i = \frac{a_i}{a_i + a_j}$ and $c(a_i) = a_i$. V_L and V_S still represent the effective wages given v_L and v_S. Assuming symmetric agents, $a_i = a_j = a$ and agents select equilibrium effort

$$a^* = \frac{V_L - V_S}{4} \tag{26}$$

The Firm's objective remains the same: anticipating agents' optimal effort choices and Stage 2 decisions, choose a wage schedule v_L and v_S which maximizes two-stage profit. The Tullock lottery promotion mechanism alters results dramatically. In this version of the game, the Firm possesses no incentive to increase the wage dispersion. The marginal benefit of an increase in v_L is

$$\frac{\partial \pi}{\partial v_L} = \frac{1}{2} q - q \tag{27}$$

which is always negative. As before, increasing v_S also decreases profit. The Firm always prefers setting v_L and v_S as low as possible. This implies solutions to the Firm's problem which incentivize low first-stage effort. The Firm ultimately chooses the optimal wage schedule depending on the value of second-stage project completion D. For sufficiently high values of D, the benefit of project completion outweighs the loss from setting a positive wage spread, and the firm prefers $v^*_{1,1} = (w_L, w_S)$. Otherwise, the Firm prefers wages which induce either $d^* = (1, 0)$ or $d^* = (0, 0)$.

Optimal wages under the standard Tullock lottery mechanism resemble the results from the rank-order tournament only when high degrees of noise exist. This exercise illustrates the sensitivity of optimal incentive design to the type of promotion contest. Further research should investigate other contest formats including all-pay contests or variations on the Tullock contest in which Firms prefer positive wage spreads.

5 Discussion

In this paper, I study the design of a dynamic internal promotion tournament in which peers compete for the right to earn the highest wage and then work together on a joint project. Firms manipulate relative wages to induce competition between peers and adjust absolute wages to facilitate coordination across levels of organizational hierarchy. When individual worker output clearly signals worker effort, firms prefer significantly unequal incentive schemes in which all employees earn a substantial wage. This incentive scheme guarantees future team effectiveness while maximizing each individual's ambition for upward mobility. In other cases, firms prefer

winner-take-all incentive schemes or low wages for all workers. Some settings involving noisy output and low strategic uncertainty demand an optimal incentive mechanism consisting of perfectly equal wages. The relationship between the value of joint collaboration and the nature of individual output greatly impacts optimal wage design.

This model establishes a framework for further studying the dynamics of internal promotions. Future work should incorporate standard extensions of tournament theory including industrial politics and influence activities. For example, how does sabotage in a tournament affect the future interaction of peers after one worker earns promotion? Alternatively, how do workers and firms respond when firms promote based on influence activities rather than sheer effort and output? In particular, if passed-over workers observe the means by which the promotee earned promotion, they may condition future behavior on this information. Workers may punish promotees who climb the corporate ladder by unethical or unfair means. Integrating industrial politics or influence activities into the basic setting inspires many interesting lines of inquiry.

A different avenue of future work could investigate different post-tournament environments. In this paper, I focus on post-tournament coordination. Future work could instead explore post-tournament cooperation in a prisoner's dilemma or public goods game. Given the results of this paper, it may be particularly interesting to focus on post-tournament games with multiple equilibria and explore how tournament competition interacts with strategic uncertainty and equilibrium selection. Future work should also manipulate the degree of post-tournament interaction. In some organizational structures, recent promotees work closely with former peers, while in other settings, different levels of the corporate hierarchy rarely interact. Extensions of the model should consider the effects of organizational structure on tournament competition and post-tournament interaction.

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Appendix

A.1. Proof of No Asymmetric Equilibrium Effort

I first solve the general agent's problem without assuming a symmetric equilibrium. Agent i maximizes expected utility given some V_L, V_S and a_j:

$$\max_{a_i} P_i V_L + (1 - P_i)V_S - c(a_i) \tag{28}$$

where P_i represents the probability that agent i wins promotion. In a rank-order tournament,

$$P_i = Prob(y_i > y_j) = Prob(a_i + \varepsilon_i > a_j + \varepsilon_j) = Prob(\varepsilon_j - \varepsilon_i < a_i - a_j) = F(a_i - a_j) \tag{29}$$

and $\frac{\partial F(a_i - a_j)}{\partial(a_i - a_j)} \equiv f(a_i - a_j)$. From the FOC, agent i chooses a_i such that

$$\begin{aligned} &\frac{\partial F(a_i - a_j)}{\partial a_i}(V_L - V_S) \\ &= \frac{\partial F(a_i - a_j)}{\partial(a_i - a_j)}(V_L - V_S) \\ &= f(a_i - a_j)(V_L - V_S) = \frac{\partial c(a_i)}{\partial a_i} \end{aligned} \tag{30}$$

Now consider agent j's problem. In the two-agent tournament, the probability that j wins promotion equals the probability that i loses. Agent j maximizes expected utility:

$$\max_{a_j}(1 - F(a_i - a_j))V_L + F(a_i - a_j)V_S - c(a_j) \tag{31}$$

From the FOC, agent j chooses a_j such that

$$\begin{aligned} &-\frac{\partial F(a_i - a_j)}{\partial a_j}V_L + \frac{\partial F(a_i - a_j)}{\partial(a_j)}V_S \\ &= -\frac{\partial F(a_i - a_j)}{\partial(a_i - a_j)}(-1)V_L + \frac{\partial F(a_i - a_j)}{\partial(a_i - a_j)}(-1)V_S \\ &= \frac{\partial F(a_i - a_j)}{\partial(a_i - a_j)}(V_L - V_S) \\ &= f(a_i - a_j)(V_L - V_S) = \frac{\partial c(a_j)}{\partial a_j} \end{aligned} \tag{32}$$

which exactly matches the optimality condition for agent i. I now prove that no asymmetric equilibrium exists given V_L and V_S.

Proof. Let a_i^* and a_j^* denote equilibrium effort. WLOG, suppose $a_i^* > a_j^* \geq 0$. $a_i^* > 0$ implies

$$f(a_i^* - a_j^*)(V_L - V_S) = \frac{\partial c(a_i^*)}{\partial a_i^*} \tag{33}$$

Given $c(a_i)$ convex, $a_i^* > a_j^*$ implies $\frac{\partial c(a_i^*)}{\partial a_i^*} > \frac{\partial c(a_j^*)}{\partial a_j^*}$. From optimality for j, this implies

$$f(a_i^* - a_j^*)(V_L - V_S) > \frac{\partial c(a_j^*)}{\partial a_j^*} \tag{34}$$

which suggests agent j can increase utility by increasing a_j^* until $a_j^* = a_i^*$. Therefore, the proposed asymmetric effort choices cannot be an equilibrium.

□

A.2. Optimal Wages Given $\gamma = 4$, $m = 2$ $\alpha = \frac{1}{2}$

Assume $c(a) = \frac{1}{2\gamma}a^2$ where $\gamma > 0$ is a constant. Then $\frac{\partial c(a)}{\partial a} = \frac{a}{\gamma}$. Also assume ε_i and ε_j are identically distributed such that $(\varepsilon_j - \varepsilon_i) \sim U[-b, b]$ for some $b > 0$. This implies $f(0) = \frac{1}{2b}$. Finally, assume $\gamma = 4$, $m = 2$ and $\alpha = \frac{1}{2}$. This implies $w_S = 4$ and $w_L = 8$. Given D and b, an SPE specifies the Firm's optimal wage schedule $v^* = (v_L^*, v_L^*)$, and each agent's optimal effort a^* and second-stage work decisions d_L^* and d_S^* for every possible wage.

All SPE involve the following Stage 2 agent strategy: $d_L^* = 1$ if $v_L \geq 8$ or $v_L \geq 4$ and $d_S^* = 1$. $d_L^* = 0$ if $v_L < 4$. Likewise, $d_S^* = 1$ if $v_S \geq 8$ or $v_S \geq 4$ and $d_L^* = 1$. $d_S^* = 0$ if $v_S < 4$.

Given wages v_L and v_S which imply effective wages V_L and V_S, agents choose a^* such that

$$a^* = \frac{2(V_L - V_S)}{b} \tag{35}$$

and

$$c(a^*) = \frac{(V_L - V_S)^2}{2b^2} \tag{36}$$

The Firm first identifies the optimal wages which induce each possible Stage 2 outcome. This involves solving a constrained optimization problem with artificial constraints on v_L and v_S. Then the Firm compares profit from each of these wage schedules and selects the one which offers the highest profit. For example, the firm determines the optimal wage schedule which induces $d = (1, 1)$ and $q = 1$. I can now express effective wages in terms of actual wages. I substitute for a as a function of actual wages to satisfy the IC constraint. At the profit-maximizing wage schedule, IR binds with equality. For this particular Stage 2 outcome, the Firm solves the problem:

$$\max_{v_L, v_S} \quad 2\left(\frac{2(v_L - v_S)}{b}\right) + D - v_L - v_S \quad \text{s.t.} \tag{37}$$

$$\frac{1}{2}(v_L + v_S - 4) = \frac{(v_L - v_S)^2}{2b^2} \quad (IR) \tag{38}$$

$$v_L \geq 8 \quad v_S \geq 4 \tag{39}$$

First, consider when $2 < b < 4$. From Proposition 1, we know the Firm always prefers increasing v_L in this range. Furthermore, for these particular parameters, $\frac{1}{2}\gamma = 2$. If $b > 2$, we know from

Proposition 3 that the constraints on v_L and v_S bind for each of the Firm's artificially constrained optimization problems. Therefore, the optimal wage schedule always involves setting v_S as low as possible and setting v_L such that the IR constraint binds with equality. For example, the optimal wage schedule which induces $d = (1, 1)$ involves setting $v_S = 4$ and finding v_L such that

$$\frac{1}{2}v_L = \frac{(v_L - 4)^2}{2b^2} \quad (IR) \tag{40}$$

The optimal wage schedule which induces $d = (1, 1)$ is $v^*_{1,1} = (4 + \frac{b^2+b\sqrt{b^2+16}}{2}, 4)$. This yields expected profit of $\pi^*_{1,1} = 2(b+\sqrt{b^2+16})+D-(8+\frac{b^2+b\sqrt{b^2+16}}{2})$. The Firm repeats this procedure for each corresponding Stage 2 outcome. The optimal wage schedule which induces $d = (1, 0)$ is $v^*_{1,0} = (4 + 2b^2, 0)$. This yields expected profit of $\pi^*_{1,0} = 4b + \frac{1}{2}D - (b^2 + 2)$. The optimal wage schedule which induces $d = (0, 0)$ is $v^*_{0,0} = (8, 0)$. This yields expected profit of $\pi^*_{0,0} = \frac{8}{b} + \frac{1}{4}D - 2$.

In the range of values which induce multiple Stage 2 NE ($4 \leq v_L < 8$ and $v_s \geq 4$), agents may choose $d^* = (1, 1)$ in which case the optimal wage is $v^*_M = (8, 4)$. This yields expected profit of $\pi^*_M = \frac{8}{b} + D - 12$. Notice this wage schedule is always dominated by $v^*_{1,1} = (4+\frac{b^2+b\sqrt{b^2+16}}{2}, 4)$. Agents may instead coordinate on $d^* = (0, 0)$ in which case the optimal wage is still $v^*_M = (8, 4)$. This yields expected profit of $\pi^*_M = \frac{4}{b} + \frac{1}{4}D - 3$. Notice, this wage schedule is always dominated by $v^*_{0,0} = (8, 0)$. Finally, agents may coordinate on the mixed strategy equilibrium $d = (\delta_L = \frac{8-v_S}{v_S}, \delta_L = \frac{8-v_L}{v_L})$. Under this NE, for any v_L and v_S, $V_L = V_S$ which implies $a = 0$. Profits equal $\pi^*_M = \frac{16}{v_L v_s}(D - v_L - v_S)$. The wage schedule which maximizes this profit involves $v_L = v_S$. If $D \geq \frac{16}{3}$, $v^*_M = (4, 4)$ and $\pi^*_M = D - 8$, but this is dominated by $v^*_{1,1} = (4 + \frac{b^2+b\sqrt{b^2+16}}{2}, 4)$. If $D < \frac{16}{3}$, $v^*_M = (8, 8)$ and $\pi^* = \frac{1}{4}D - 4$, but this is dominated by $v^*_{0,0} = (8, 0)$.

Given D and b, the firm ultimately chooses the profit-maximizing wage schedule from among the undominated options. The firm chooses $v^*_{1,1}$ (red region in Figure 1) if $\pi^*_{1,1} \geq \pi^*_{1,0}$ (above red line in Figure 1) and $\pi^*_{1,1} \geq \pi^*_{0,0}$ (above blue dashed line in Figure 1). The firm chooses $v^*_{1,0}$ (green region in Figure 1) if $\pi^*_{1,0} \geq \pi^*_{1,1}$ (below red line in Figure 1) and $\pi^*_{1,0} \geq \pi^*_{0,0}$ (above green line in Figure 1). Finally, the firm chooses $v^*_{0,0}$ (white region in Figure 1) if $\pi^*_{0,0} \geq \pi^*_{1,1}$ (below blue dashed line in Figure 1) and $\pi^*_{0,0} \geq \pi^*_{1,0}$ (below green line in Figure 1).

When $2 < b < 4$, three SPE exist in each colored region of Figure 1 because agents are indifferent when $4 \leq v_L < 8$ and $v_S \geq 4$. Explicitly, L is indifferent between $d^*_L = 1$, $d^*_L = 0$ and $d^*_L = \delta_L = \frac{8-v_S}{v_S}$, and S is indifferent between $d^*_S = 1$, $d^*_S = 0$ and $d^*_S = \delta_S = \frac{8-v_L}{v_L}$.

SPE in each region are characterized by the following optimal wage schedule and first-stage effort:

- Red Region: Wage schedule $v^* = (4 + \frac{b^2+b\sqrt{b^2+16}}{2}, 4)$ and effort $a^* = b + \sqrt{b^2 + 16}$
- Green Region: Wage schedule $v^* = (4 + 2b^2, 0)$ and effort $a^* = 2b$
- White Region: Wage schedule $v^* = (8, 0)$ and effort $a^* = \frac{4}{b}$

Now consider the case when $b \geq \gamma$. The Firm prefers to set v_L and v_S both as low as possible. In this case, $v^*_{1,1} = (8, 4)$ which yields expected profit $\pi^*_{1,1} = \frac{16}{b} + D - 12$. $v^*_{1,0} = (8, 0)$ which yields expected profit $\pi^*_{1,0} = \frac{8}{b} + \frac{1}{2}D - 4$. $v^*_{0,0} = (0, 0)$ which yields expected profit $\pi^*_{0,0} = \frac{1}{4}D$.

If $4 \leq v_L < 8$ and $v_s \geq 4$, agents may choose $d^* = (1, 1)$ in which case the optimal wage is $v^*_M = (4, 4)$. This yields expected profit of $\pi^*_M = D - 8$ which dominates $v^*_{1,1}$. Agents may coordinate on the mixed strategy equilibrium $d = (\delta_L = \frac{8-v_S}{v_S}, \delta_L = \frac{8-v_L}{v_L})$. If $D \geq \frac{16}{3}$, the

wage schedule which maximizes this profit is $v_M^* = (4, 4)$. This yields profit $\pi_M^* = D - 8$ which dominates $v_{1,1}^*$. If $D < \frac{16}{3}$, $v_M^* = (8, 8)$ and $\pi^* = \frac{1}{4}D - 4$, but this is dominated by $v_{0,0}^*$.

Given the Stage 2 agent's strategy described above including $d_L^* = d_S^* = 1$ or $d_L^* = \delta_L = \frac{8-v_S}{v_S}$ and $d_S^* = \delta_S = \frac{8-v_L}{v_L}$ if $4 \leq v_L < 8$ and $v_s \geq 4$, SPE wage schedule and first-stage effort in each region in Figure 2 are characterized by:

- Red Region: Wage schedule $v^* = (4, 4)$ and effort $a^* = 0$
- Green Region: Wage schedule $v^* = (8, 0)$ and effort $a^* = \frac{4}{b}$
- White Region: Wage schedule $v^* = (0, 0)$ and effort $a^* = 0$

Agents may instead coordinate on $d^* = (0, 0)$ when $4 \leq v_L < 8$ and $v_s \geq 4$ in which case the optimal wage is still $v_M^* = (4, 4)$. This yields expected profit of $\pi_M^* = \frac{1}{4}D - 2$. Notice, this wage schedule is always dominated by $v_{0,0}^*$. SPE wage schedule and first-stage effort in each region in Figure 3 are characterized by:

- Red Region: Wage schedule $v^* = (8, 4)$ and effort $a^* = \frac{8}{b}$
- Green Region: Wage schedule $v^* = (8, 0)$ and effort $a^* = \frac{4}{b}$
- White Region: Wage schedule $v^* = (0, 0)$ and effort $a^* = 0$

For the parametric assumptions above, this fully characterizes equilibria of the two-stage game.